Phi-Law / LIM: Axioms A0–A3 and Formal Framework

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Part I: The Axioms

The framework rests on four foundational axioms (A0–A3), stated in their original Norwegian with English translation. These are not derived — they are the starting commitments from which the formal structure is built.

A0 *Ingen er utan blir.*

No being without becoming.

> Existence is not a static state. Every existing thing is already in the process of becoming something else.

A1 *Ingen blir utan er.*

No becoming without being.

> Transformation requires a substrate. Process without structure collapses into noise.

A2 *Framleis er meir grunnleggjande enn identitet.*

Continuity is more fundamental than identity.

> The process (F) is definable without reference to the fixed point (I*). The fixed point requires F to exist. Therefore process precedes identity — ontologically, not temporally.

A3 *Alt som eksisterer, eksisterer gjennom transformasjon.*

Everything that exists, exists through transformation.

> There is no existence outside process. Identity is a trace of filtered transformation, not its precondition.

Part II: The Coherence Metric tau

A2 claims that process precedes identity. The metric tau makes this measurable. It is computed from the hidden state matrix of a language model via singular value decomposition.

Definition

Given hidden state matrix H (N x d):

1. `Compute SVD: H = U S V^T. Let s_1, ..., s_r be the singular values.`
2. `Normalise squared singular values: p_i = s_i^2 / sum(s_j^2)`
3. `Spectral entropy: H_spectral = -sum(p_i * log(p_i))`
4. `Effective rank: r_eff = exp(H_spectral)`
5. `tau = r_eff / r_max where r_max = min(N, d)`

tau is dimensionless, bounded in [0,1], and measures how evenly the singular value spectrum is distributed. tau = 1: uniform distribution (maximum entropy). tau -> 0: rank-1 collapse.

Part III: The Coherence Bounds (Theorems 1 and 3)

Theorem 1 — Lower bound via Euler-Mascheroni

For a minimal self-dual operator L on a Hilbert space H with involution J (JLJ = L^-1), the coefficient of the logarithmic divergence in the spectral zeta function zeta_L(s) equals the Euler-Mascheroni constant gamma.

```
zeta_L(s) = sum lambda_n^(-s) for Re(s) > 1
```

Eigenvalues come in pairs (lambda, 1/lambda) from the self-duality condition. The regularisation of the divergence between the discrete sum and its continuous integral yields gamma. This gives the lower coherence bound:

```
tau_min = exp(-gamma) = 0.5615...
```

Below tau_min: the system has lost the memory of its ground state. It drifts toward entropic collapse.

Theorem 3 — Upper bound via Apery's constant

For the same operator L, the spectral zeta function at s=3 evaluates to Apery's constant zeta(3). This is the third moment of the spectrum — measuring the weight of the high-energy tail.

```
zeta_L(3) = sum lambda_n^(-3) -> zeta(3) = 1.2020569...
```

This gives the upper coherence bound:

```
tau_max = 1/zeta(3) = 0.8319...
```

Above tau_max: the system is too rigid. The high-energy modes dominate and the system loses adaptive capacity.

The Goldilocks interval

A system maintaining identity through transformation must satisfy:

```
tau_min < tau < tau_max
exp(-gamma) < tau < 1/zeta(3)
0.5615 < tau < 0.8319
```

Part IV: Formal Derivation of A2 (Banach Fixed-Point Theorem)

A2 states that continuity is more fundamental than identity. This is formalised as follows.

The Framleis operator

Let S = [tau_min, tau_max] with the standard metric. Define the operator F: S -> S by:

```
F(tau; sigma) = (1 - alpha) * tau + alpha * sigma
```

where sigma is the local incoming signal at each step. F is a local rule: it requires only the current state (tau), the filtering rate (alpha), and the local signal (sigma). It has no knowledge of the fixed point.

Theorem (Banach, 1922)

Let (S, d) be a complete metric space and $F: S \to S$ a contraction with factor $k < 1$. Then there exists a unique fixed point I^* such that $F(I^*) = I^*$, and for all τ_0 in S: $F^n(\tau_0) \to I^*$.

Derivation

Step 1. F is a contraction.

```
|F(tau_1; sigma) - F(tau_2; sigma)| = (1 - alpha) * |tau_1 - tau_2|
```

Since (1 - alpha) < 1, F is a contraction with factor k = 1 - alpha.

Step 2. The fixed point I* emerges when sigma stabilises.

```
When sigma -> sigma* (stable input), F(I*; sigma*) = I* solves as:
(1 - alpha) * I* + alpha * sigma* = I*
=> I* = sigma*
```

I* is not known in advance. It is what crystallises when the rule runs long enough.

Step 3. Ontological priority.

a) F is definable without reference to I*. b) I* is defined as the limit of F under stable sigma — it requires F to exist. c) F exists and is contractive independently of whether I* has been reached. d) Identity I* is a product of process F, not a starting point.

Since F exists independently of I*, but I* exists only as the limit of F, process (Framleis = F) is ontologically more fundamental than identity (I*). This is A2.

Note on the empirical side: the scaling law tau ~ 0.10 x N^0.48 is curve-fitted to three models (GPT-2, Phi-2, Mistral-7B) and is not derived from the above. The formal framework is Parts I–IV. The scaling law is an observation.