Framleis Law

Peer Review Draft: Mathematical Structure, Open Assumptions, and Falsification Path

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Abstract

This manuscript is submitted for technical and mathematical peer review, not for publication. The purpose is to identify errors, hidden assumptions, missing proofs, unjustified generalizations, weak derivations, and alternative interpretations before any public dissemination.

The proposed Framleis Law studies adaptive systems through spectral rank density τ and a contraction operator

$$F(\tau; \sigma^*) = (1 - \alpha)\tau + \alpha\sigma^*.$$

The fixed-point and convergence results are elementary. The stronger claims concerning universal constants, especially

$$\alpha_{\text{opt}} = 1 - e^{-\gamma}$$

and the proposed interval

$$\left[e^{-\gamma}, \frac{1}{\zeta(3)}\right],$$

are explicitly presented as hypotheses or conjectures requiring review and empirical validation.

Status: Peer review draft. Not for citation as final work.

Requested review focus: mathematics, hidden assumptions, falsifiability, notation, and whether any claim is overstated.

Contents

1 Questions for Reviewers

Review Question 1.1. *Is spectral rank density τ a defensible order parameter for comparing adaptive systems across domains?*

Review Question 1.2. *Does the Banach contraction framing support only local convergence, or can it support any broader universality claim?*

Review Question 1.3. *Is the proposed adaptation constant*

$$\alpha_{\text{opt}} = 1 - e^{-\gamma}$$

mathematically justified, or does it require an additional unstated assumption?

Review Question 1.4. *Is the proposed upper bound*

$$\tau_{\max} = 1/\zeta(3)$$

connected rigorously enough to spectral instability, random matrix theory, or gradient variance?

Review Question 1.5. *Which statements should be downgraded from theorem to conjecture or hypothesis?*

Review Question 1.6. *Are the falsification criteria strong enough to make the framework scientifically testable?*

2 What Is Actually Proved

The following claims are currently mathematical results:

1. The Framleis operator has a fixed point at $\tau^* = \sigma^*$.
2. The iteration converges exponentially when $0 < \alpha < 1$.
3. The spectral rank density definition is well-defined for finite matrices with nonzero singular spectrum.

The following claims are not yet proved:

1. That $\alpha_{\text{opt}} = 1 - e^{-\gamma}$ is universal.
2. That $[e^{-\gamma}, 1/\zeta(3)]$ is a universal optimal interval.
3. That τ predicts semantic or adaptive coherence across domains.
4. That the Lyapunov-style expression proposed here captures actual stability across systems.

3 Definitions

Definition 3.1 (Spectral probabilities)**.** *Let $W \in \mathbb{R}^{m\times n}$ have singular values*

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0.$$

Define

$$p_i = \frac{\sigma_i^2}{\sum_{j=1}^{r} \sigma_j^2}.$$

Definition 3.2 (Spectral entropy)**.**

$$H(W) = -\sum_{i=1}^{r} p_i \ln p_i.$$

Definition 3.3 (Effective rank)**.**

$$r_{\text{eff}}(W) = \exp(H(W)).$$

Definition 3.4 (Spectral rank density)**.**

$$\tau(W) = \frac{r_{\text{eff}}(W)}{\min(m, n)}.$$

Definition 3.5 (Framleis operator)**.**

$$F(\tau; \sigma^*) = (1 - \alpha)\tau + \alpha\sigma^*,$$

where

$$0 < \alpha < 1.$$

4 Axioms Under Review

Axiom 4.1 (Local controllability)**.** *Adaptive systems can be updated locally without complete global knowledge.*

Axiom 4.2 (Emergent global structure)**.** *Local update rules can generate global structure without explicit global design.*

Axiom 4.3 (Spectral coherence)**.** *Spectral rank density τ can represent system-wide coherence.*

Axiom 4.4 (Contraction dynamics)**.** *Adaptive movement toward coherence can be modeled as a contraction.*

Remark 4.1. These are not all equally established. Axiom 4.4 is mathematically clean once accepted. Axiom 4.3 is the most important and most vulnerable assumption.

5 Fixed-Point Mathematics

Theorem 5.1 (Fixed point)**.** *For*

$$F(\tau;\sigma^*) = (1-\alpha)\tau + \alpha\sigma^*,$$

the fixed point is

$$\tau^* = \sigma^*.$$

Proof. At the fixed point,

$$\tau^* = (1-\alpha)\tau^* + \alpha\sigma^*.$$

Thus

$$\alpha\tau^* = \alpha\sigma^*.$$

Since $\alpha > 0$,

$$\tau^* = \sigma^*.$$

□

Theorem 5.2 (Exponential convergence)**.** *If*

$$\tau_{t+1} = F(\tau_t;\sigma^*),$$

then

$$|\tau_t - \sigma^*| = (1-\alpha)^t|\tau_0 - \sigma^*|.$$

Proof.

$$\tau_{t+1} - \sigma^* = (1-\alpha)(\tau_t - \sigma^*).$$

Iterating gives the result. □

6 Conjectures Requiring Review

Conjecture 6.1 (Euler–Mascheroni adaptation rate). *The optimal adaptation strength is*

$$\alpha_{\text{opt}} = 1 - e^{-\gamma} \approx 0.4389.$$

Review request: identify whether this follows from a valid variational or information-theoretic argument, or whether the use of γ is unjustified.

Conjecture 6.2 (Goldilocks interval). *The optimal adaptive interval is*

$$\tau \in \left[e^{-\gamma}, \frac{1}{\zeta(3)}\right] \approx [0.5615, 0.8319].$$

Review request: assess whether either boundary has a defensible derivation, and whether the two constants belong in the same framework.

Conjecture 6.3 (Coherence prediction). *Systems inside the proposed interval should show greater adaptive stability than systems outside it.*

7 Proposed Stability Model

Define

$$R(\tau) = (\tau - e^{-\gamma}) \left(\tau - \frac{1}{\zeta(3)}\right).$$

A proposed Lyapunov-style diagnostic is

$$\lambda(\tau) = \ln |1 - R(\tau)|.$$

This section is speculative.

At the proposed boundaries:

$$R(e^{-\gamma}) = 0, \qquad R(1/\zeta(3)) = 0.$$

Therefore:

$$\lambda(e^{-\gamma}) = 0, \qquad \lambda(1/\zeta(3)) = 0.$$

Review request: determine whether this expression has genuine dynamical meaning or is only a constructed boundary condition.

8 Classical Anchors

8.1 Khinchin

Khinchin's continued fraction result is used only as an analogy for local iteration producing global statistical regularity.

Reviewer warning: this does not prove Framleis Law.

8.2 Bayes

Bayesian updating can resemble convex interpolation:

$$\mathbb{E}[\theta|D] = (1-\lambda)\mathbb{E}[\theta]_{\text{prior}} + \lambda\mathbb{E}[\theta]_{\text{data}}.$$

This resembles:

$$F(\tau;\sigma^*) = (1-\alpha)\tau + \alpha\sigma^*.$$

Reviewer warning: resemblance is not derivation.

8.3 Schrödinger

The Schrödinger equation preserves norm under unitary evolution:

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi.$$

Reviewer warning: this is only a constrained-evolution analogy unless a stronger mapping is supplied.

9 Empirical Tests Needed

1. Test τ on transformer hidden states.
2. Compare coherent, random, repetitive, and adversarial outputs.
3. Compare τ against effective rank, entropy, participation ratio, and PCA variance.
4. Test whether the proposed interval appears without fitting.
5. Run Marchenko–Pastur random matrix controls.
6. Test perturbation decay inside and outside the proposed interval.

10 Falsification Criteria

The framework should be rejected or revised if:

1. τ does not outperform simpler spectral metrics.
2. The proposed interval does not appear in controlled experiments.
3. The constants cannot be derived without hidden assumptions.
4. Random matrices appear equally supportive of the framework.
5. Stability does not differ inside versus outside the proposed interval.

11 Known Weak Points

1. The Euler–Mascheroni derivation is incomplete.
2. The $\zeta(3)$ upper bound is not yet rigorously connected to adaptive instability.
3. Cross-domain universality is unproven.
4. The Lyapunov expression may be constructed rather than derived.
5. The biological, economic, and social extensions require domain-specific validation.

12 Conclusion

This draft asks whether Framleis Law can be made mathematically and empirically defensible.

The fixed-point core is simple and valid.

The universality claim remains open.

The purpose of this peer review draft is to expose the weakest assumptions before the work is developed further.

References

References

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